

Harmonic Tidal Analysis along T/P Tracks in China Seas and Vicinity

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Abstract. More than 300 cycles of TOPEX/POSEIDON altimeter data covering China Seas and the West Pacific Ocean are adopted to along track harmonic tidal analyses. Special concerns are placed on the tidal aliasing problem caused by the T/P sampling rate, and a criterion for separation of constituents expressed in terms of the condition number of the normal matrix of the least squares estimator is presented. The accuracy and reliability of the results are assessed by intercomparison between the results of both ascending and descending ground tracks at crossovers, and by the comparison with in situ data of tide gauges. The comparisons indicate that the harmonic constants derived from T/P track observations are more accurate than the corresponding ones of existing global ocean models, especially, over the Yellow Sea, the East China Sea and the South China Sea, characterized as continental shelves seas and semiencloded sea, where the quality of tide information is evidently improved. The root sum squares (RSS) of differences between the parameters of 4 principal tides resulted from harmonic analyses along T/P tracks and the corresponding ones from the tidal tables of hydrography of the navy in the study area is of 7 cm in magnitude. Considering the fact that the qualities of the tidal tables are not precise enough, the precision indicated by the RSS mentioned above shows an improvement on the tide recovery over coastal waters by the method used in this paper. However the along track solutions can satisfy the need for providing open boundary values to develop regional ocean tide models with assimilation techniques.

Keywords. Principal constituents; Along track data; Harmonic analysis; China Seas

1 Introduction

Since the launch of TOPEX/POSEIDON (T/P), considerable efforts have been made to estimate the ocean tides on the global scale. At present, the accuracies of the tide models derived from T/P data have been improved to 2-3 cm RMS over the open oceans [Shum et al, 1997]. To overcome the aliasing constraints with short time span of observation, it has been necessary to bin the data on boxes of several degrees in order to sample a wider range of phase [LeProvost, 1995]. In terms of this approach, small scale features of tidal field are made smooth down, therefore, most of the state of the art global tide models are not accurate enough over continental shelves and semiencloded seas. A challenge is given to geodetists and oceanographers to improve the accuracy of the models over these regional areas.

With the availability of a long data record of accurate sea surface height measurements, it is now possible to estimate the ocean tide along the ground tracks of T/P. This had been done from over 4.5 years of data using both response and harmonic analyses [Tierney et al, 1998].

In this paper, based on theoretical analysis of the aliasing problem, we estimate tides along the ground tracks of T/P using harmonic analysis from 302 cycles of T/P data over China Sea and vicinity. By intercomparison of both ascending and descending results at crossover locations and by comparisons with the ground truth, the accuracies of the estimated results are assessed.

2 Aliasing Periods and a Decorrelation Criterion in Harmonic Analysis

Because the altimetry satellites have repeat periods of a few days or more, aliasing of the diurnal and semidiurnal tides into long period signals is an inherent property of the sampling of tidal signals in satellite altimetry. Aliasing induces tidal correlations. Therefore, observation time span of much longer than that of tide gauges is needed to

aliasing period, by Rayleigh's rule, the time span T is needed for the separation of constituent i from constituent j :

$$T \geq \left| \frac{T_{ai}T_{aj}}{T_{ai} - T_{aj}} \right| \quad (1)$$

This criterion is derived based on the principle of signal analysis. At present, the least squares method is considered as a standard approach for harmonic tidal analysis, therefore, the separability of constituents to be estimated, mainly depends on the condition number of the normal matrix of the least squares estimates.

The tidal height at a given location is written as:

$$\begin{aligned} h(t) &= \bar{h} + \sum_{j=1}^m f_j H_j \cos(\sigma_j t + V_{0j} + u_j - G_j) \\ &= \bar{h} + \sum_{j=1}^m [f_j \cos(\sigma_j t + V_{0j} + u_j) H_j^C \\ &\quad + f_j \sin(\sigma_j t + V_{0j} + u_j) H_j^S] \end{aligned} \quad (2)$$

Where m is the number of constituents included in the tidal height expression, $\bar{h}$ denotes the height of the mean sea level relative to the Mean Sea Surface (MSS) provided by the MGDRs. σ_j and V_{0j} are the angular velocity and the phase at a given reference epoch respectively. f_j and u_j are the nodal factors. H_j and G_j are the harmonic constants, i.e., amplitude and phase lag, and H_j^C, H_j^S are in-phase part and quadrature part of the constants.

Thus, the observation equations for a sea surface height series can be written as:

$$L = BX + A \quad (3)$$

Where L denotes the vector of observations, A the vector of errors, and X the vector of unknowns which takes the form of:

$$X = (\bar{h} \ H_1^C \ H_1^S \ H_2^C \ H_2^S \ \dots \ H_m^C \ H_m^S)^T \quad (4)$$

The matrix B can be written as:

$$B = \begin{bmatrix} 1 & C_{11} & S_{11} & C_{12} & S_{12} & \dots & C_{1m} & S_{1m} \\ 1 & C_{21} & S_{21} & C_{22} & S_{22} & \dots & C_{2m} & S_{2m} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & C_{n1} & S_{n1} & C_{n2} & S_{n2} & \dots & C_{nm} & S_{nm} \end{bmatrix} \quad (5)$$

Where n is the number of observations, ($i=1,2,\dots,n$, $j=1,2,\dots,m$), C_{ij} and S_{ij} denote $f_j \cos(\sigma_j t_i + V_{0j} + u_j)$, and $f_j \sin(\sigma_j t_i + V_{0j} + u_j)$ respectively.

Therefore, the unknown parameter X can be estimated by the least squares method through solving the normal equation:

$$NX = B^T L \quad (6)$$

Where $N = B^T B$

According to Rayleigh's rule, it is necessary to use at least 9 years of T/P data for a reliable harmonic estimate of tides.

With the above expressions, we propose a common criterion for a reliable estimate as:

$$\text{cond}(N) = \|N\|_2 \|N^{-1}\|_2 = \frac{|\lambda_{\max}|}{|\lambda_{\min}|} \leq \text{Limen} \quad (7)$$

Where $\text{cond}(\cdot)$ denotes condition number, $\lambda_{\max}$ and $\lambda_{\min}$ the maximum and minimum eigenvalues of matrix N respectively. Here, we take 10 as an given limen.

The criterion given above is suitable not only for along track tidal analysis, but also for the tidal estimates at crossovers or in bins. Obviously, the solubility of normal equation is equivalent to the separability of the tidal frequencies from sampling series in our studied problem. Therefore, we can identify if the constituents can be separated or not according to a theoretical standard instead of an empirical one.

Supposing that 10 major constituents (Sa, SSa, Q1, O1, P1, K1, N2, M2, S2, K2) are included in our tidal height expression, the condition numbers of matrix N for various time spans indicated by the number of cycles used in solution with T/P sampling rate are computed and plotted as Fig. 1.

Fig. 1 shows that if we have about 140 cycles of T/P data, the condition number of matrix N for point tidal analysis will be less than the given limen of 10, which happens to agreement in the sense of cycles used in the analysis presented by Tierney et al [1998]. If the observation time span is shorter

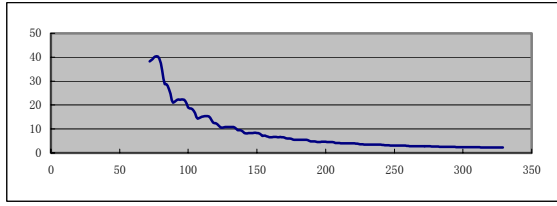


Fig. 1 The condition numbers of matrix N vs. cycles of T/P data used in solution, here abscissa denotes cycle numbers, and ordinate condition number of matrix N

than 2 years, the corresponding condition number will be too large to do a reliable estimate, for example, when only 1 year T/P data (corresponding to 36 cycles) are used for harmonic tidal analysis, the condition number will increase to a magnitude of millions. In this case, the binning approaches should be adopted. In fact, up to now, longer than 9 years of T/P data have been available for tides recovering. Therefore it is possible to do along track harmonic tidal analysis both in open oceans and over shallow waters.

3 Data and Methodology

The study area covers $100^{\circ} \sim 150^{\circ}$ E, $0^{\circ} \sim 50^{\circ}$ N. The ground tracks are shown in Fig. 2. About 8 years of T/P data from pass 2 through pass 303 are used in our computations.

The altimeter ranges are corrected for the wet and dry tropospheric delays, the ionospheric delays, electromagnetic bias, tide loading (CSR3.0 model) and pole tide corrections all provided by MGDRs. The standard geophysical corrections excluding ocean tidal correction are applied to sea surface height to form residual sea surface heights for tidal analysis.

Along each track, the normal points are selected at 0.2° interval in latitude, all observations in 0.1° vicinity of the normal points are employed to form the observation equations (2). At each of the 5293 normal points, the tidal parameters are estimated based on the least squares principle.

4 Solutions and Accuracy Assessments

4.1 The Solutions

Along each of the ascending and descending ground track, the estimated harmonic constants show regular patterns, and some tidal wave features on short scale can be revealed. As examples, Fig. 3(a), and 3(b) show the estimated harmonic constants of the 5 largest constituents along pass075 profile.

Although tidal waves mostly have long wave length, on continental shelves and in semienclosed seas, the resolution of T/P tracks in cross direction may not satisfy the expression of the fine tidal structure, e.g. to diurnal and semidiurnal tides. Therefore here we only provide the amplitude map of the annual tide S_a as Fig. 4, and the map of residual mean sea surface height referenced to the OSU MSS95 model as Fig. 5

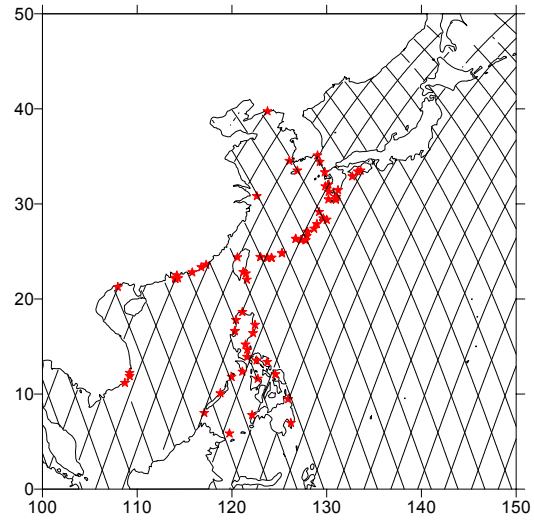


Fig. 2 distribution of T/P ground tracks and tidal gages (★) for assessment in the study area

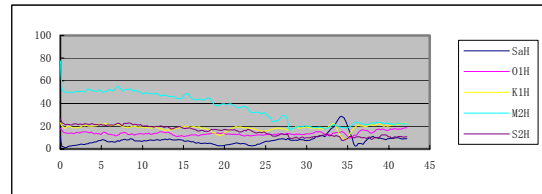


Fig. 3 (a) amplitude estimates

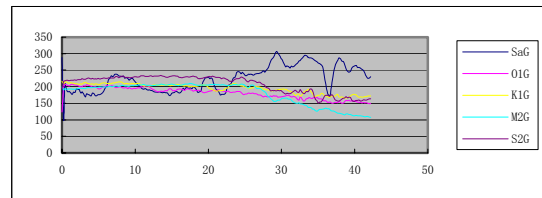


Fig. 3 (b) phase lag estimates

Fig. 3 estimated harmonic constants along pass075, here, ordinate denotes harmonic constants, i.e. amplitudes in cm or phase lags in deg., abscissa denotes latitude

Both Fig. 4 and Fig. 5 show almost the same anomalous area located in the east of Japan, where Kuroshio path pass through. Also, the tidal

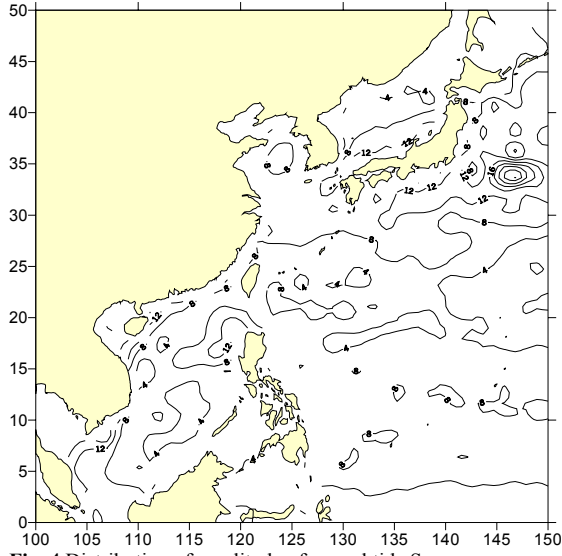


Fig. 4 Distribution of amplitude of annual tide Sa

harmonic constants derived both from ascending and descending ground tracks in this anomalous area are very different from those derived in other normal areas, which maybe caused by non-tidal dynamic oceanic signals, such as Kuroshio, and a possibility that could be trapped waves, such as internal tides.

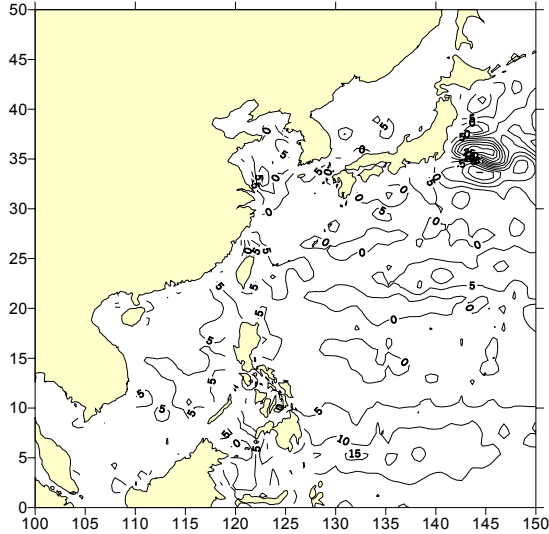


Fig. 5 The distribution of residual mean sea surface heights

4.2 Accuracy Assessment

For data comparison, the root mean squares (RMS) of difference of amplitude, phase lag, and of misfit of a constituent is defined as:

$$RMS_H = \left\{ \frac{1}{N} \sum_{i=1}^N (H_i - \bar{H}_i)^2 \right\}^{\frac{1}{2}} \quad (8)$$

$$RMS_G = \left\{ \frac{1}{N} \sum_{i=1}^N (G_i - \bar{G}_i)^2 \right\}^{\frac{1}{2}} \quad (9)$$

$$RMS = \left\{ \frac{1}{2N} \sum_{i=1}^N [(H_i^C - \bar{H}_i^C)^2 + (H_i^S - \bar{H}_i^S)^2] \right\}^{\frac{1}{2}} \quad (10)$$

Where N denotes the number of locations for comparison, the symbol with and without a bar denote the parameters in comparisons.

The root sum squares (RSS) of the 4 principal tides is defined as:

$$RSS = \left(\sum_{i=1}^4 RMS_i^2 \right)^{\frac{1}{2}} \quad (11)$$

For the comparisons in this paper, unit in centimeter for RMS_H and RMS , degree for RMS_G .

4.2.1 Standard Deviations from the Least Squares Fit

The standard deviations from the least squares fit reflect the differences level between the model tidal heights and the observed ones. Considering the high accuracy of the T/P altimeter data, we should suppose that these deviations be mainly caused by nontidal factors. The distributions of the deviations are mapped in Fig. 6.

Fig. 6 shows that in most of the study area, the standard deviations are at the 10 cm level. Near to coast, they increase to 20 cm even larger, this error magnitude is consistent with that of harmonic analysis of tidal gauge data. The anomalous distribution also occurs in the east of Japan.

4.2.2 Intercomparisons of the along Track Estimates at Crossover Locations

The differences of the parameters separately estimated from ascending and descending tracks at crossovers can provide important information for error assessment of the along track solutions. We divide our study area into 4 sub areas. They are The East China sea, The South China sea, The West Pacific ocean and the other areas. The crossover locations and sub area divisions are displayed in Fig. 7.

The comparisons of the estimated parameters at crossover locations are separately performed in each of the subdivision areas. The statistical information are plotted as Figs 8(a), 8(b), and 8(c).

Based on the statistical results, over all the 4 sub areas, the RMS of amplitude discrepancies and misfits are less than 5cm, the RMS of phase lag discrepancies are within 10° except constituent K_1 in above mentioned anomalous area. The estimated solutions have good coincidences at the crossover locations.

Some examples of the along track estimates for principal tides and their differences at crossovers in typical shallow water areas are listed in table 1, Table 1 shows that the differences between the estimated amplitudes of the principal tides from ascending and descending tracks at crossovers are less than 6 cm, and of the phase lags, are within 10° except S_2 the Beibu Gulf, because its amplitude is less than 5 cm there.

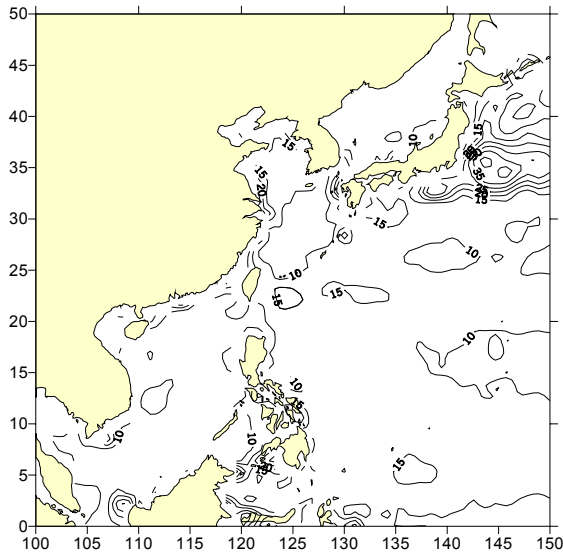


Fig. 6 the distribution of the standard deviations of the least squares fit over the study area (in cm)

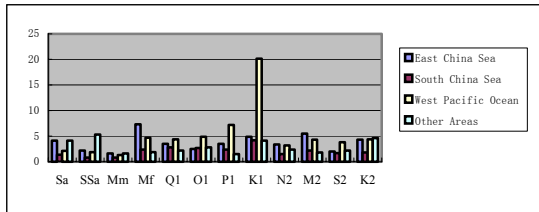


Fig. 8(a) RMS of amplitude differences

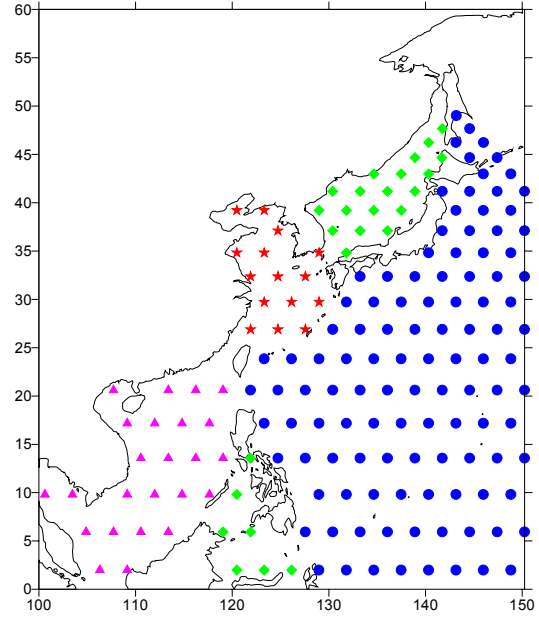


Fig. 7 The distribution of crossover locations, where the symbols $\star$, $\blacktriangle$, $\bullet$ and $\blacklozenge$ denote locations in The East China sea, The South China sea, The West Pacific Ocean and other areas respectively.

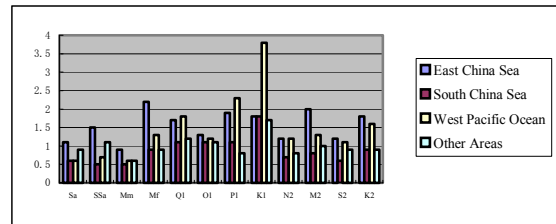


Fig. 8(b) RMS of phase lag differences

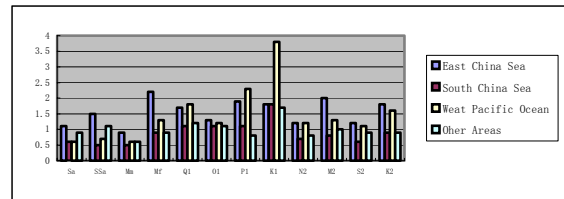


Fig. 8 (c) RMS of misfits

Fig. 8 statistical results from comparisons of along track estimates at crossovers.

Table 1. examples of along track estimates

constituents			S _a		O ₁		K ₁		M ₂		S ₂	
Harmonic constants			H	g	H	g	H	g	H	g	H	g
Longitude	120.48	Ascending	14.1	220.4	15.4	49.1	20.4	92.6	22.7	358.2	11.9	38.0
Latitude	39.21	Descending	14.6	221.6	16.6	47.2	21.7	89.9	27.9	352.0	10.6	31.3
		Differences	0.5	1.2	1.2	1.9	1.3	2.5	5.2	6.2	1.3	7.7
Longitude	107.72	Ascending	14.2	322.3	84.0	30.9	75.9	85.9	13.7	142.2	4.0	141.2
Latitude	20.61	Descending	13.1	326.0	85.4	32.1	75.7	86.8	14.7	143.0	4.3	164.3
		Differences	1.1	3.7	1.4	1.2	0.2	0.9	1.0	0.8	0.3	23.1
Longitude	123.32	Ascending	16.3	227.2	20.8	299.4	30.4	335.4	151.4	260.8	44.5	305.1
Latitude	39.21	Descending	12.2	228.7	21.6	306.8	35.3	338.3	148.0	261.8	44.8	304.5
		Differences	4.1	1.5	0.8	6.4	4.9	2.9	3.4	1.0	0.3	0.6

4.2.3 Comparisons to Tidal Gauge Data

80 tidal gauges listed in Fig. 2 are selected for error assessments., which gauges are near to T/P tracks and near coast and on islands. The RMSs and RSS for the 4 principal constituents are calculated and listed in table2.

Table 2. RMSs and RSS misfits of along track solutions to tidal gauge data

constituent	O ₁	K ₁	M ₂	S ₂
RMS	2.71	3.11	5.21	2.68
RSS	7.16			

The comparisons show that the errors of RMS of each constituent derived from along track analysis is

under 6 cm level and the corresponding RSS for the 4 principal constituents is about 7 cm.

The comparisons of the CSR3.0 tide model to the same set of tidal gauge data are performed, and the RMSs and RSS are listed in table 3.

Table 3. RMSs and RSS misfits of CSR3.0 model to tidal gauge data

constituent	O ₁	K ₁	M ₂	S ₂
RMS	8.65	11.45	35.99	15.11
RSS	41.59			

Table 2 and table 3 show that the along track solutions are more reliable than CSR3.0 models over China seas., because the along track data used in the calculations has high space resolution, and the solutions avoid binning effect. The solutions will be helpful to develop new tide models over China seas.

5 Conclusions

The criterion for separation of aliased tides from altimetry data proposed in this paper is useful to along track harmonic tidal analysis.

More than 300 cycles of T/P data are used for estimating tidal solutions over China Seas and vicinity, and the harmonic constants of the principal constituents are obtained in 5318 along track points.

Through various comparisons, the along track tidal estimates in the study area is improved to 5 cm accuracy.

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